Indian Institute of Technology Hyderabad

Department of Mathematics

Assignment 2 | Due date : 16.09.16

Date : 12.09.16 | MA 4020 : Linear Algebra | Max Marks: 30

1. For any prime p, show that $\mathbb{Z}/p\mathbb{Z}$ is a field. Why the assumption p prime is required? If p is not a prime, then what can happen? [5]

2. Prove that an arbitrary intersection of subspaces in a vector space is again a subspace. [3]

3. Let A be non-empty subset of a vector space V. Let S be a subspace of V. Prove that the following statements are equivalent: [5]

    1. A is a basis of S.
    2. Every element of S can be uniquely written as a linear combination of elements of A.
    3. A is a maximal linearly independent subset of S.
    4. A is a minimal subset of S such that $\mathrm{sp}(A) = S$.

4. Show that a finite set can never be a vector space over an infinite field. [2]

5. Show that n linearly independent vectors in $\mathbb{R}^n$ forms a basis of $\mathbb{R}^n$. If you take these n vectors as column vectors of a matrix A, then show that A is invertible. [5]

6. Show that set of all continuous functions on $[0,1]$ with values in $\mathbb{R}$ is not a finite dimensional vector space over $\mathbb{R}$. [5]

7. Prove that $\dim(L(V,W)) = \dim(V)\dim(W)$. (Hint: For every $1 \leq p \leq \dim(V), 1 \leq q \leq \dim(W)$, define [5]

$$E^{p,q}(\alpha_i) = \begin{cases} 0 & i \neq q, \\ \beta_p & i = q, \end{cases}$$

where $\{\alpha_i\}_{i=1}^{n}$ and $\{\beta_j\}_{j=1}^{m}$ are a basis of V and W, respectively.